Directions:

1. Write your name with one character in each box below.

2. Show all work. No credit for answers without work.

3. You are permitted to use one 8.5 inch by 11 inch sheet of prepared notes. No other aides are allowed.

1. [**15 points**] Given the matrix A and the reduced row echelon form of A, find bases for Col(A) and Nul(A).

$$A = \begin{bmatrix} 0 & -1 & -2 & -5 & 0 & 1 & -1 \\ 0 & -2 & -4 & -10 & -2 & 11 & 14 \\ 0 & 1 & 2 & 5 & 1 & -5 & -6 \\ 0 & 0 & 0 & 0 & 1 & -5 & -9 \end{bmatrix} \cdots \to \cdots \begin{bmatrix} 0 & 1 & 2 & 5 & 0 & 0 & 3 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

2. [**10 points**] Let V be the subspace of $\mathbb{R}^4$ containing the vectors $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}$ such that $x_1 + x_2 + x_3 + x_4 = 0$ and $x_1 - 2x_2 + 3x_3 - 4x_4 = 0$. Find a basis for V.

3. **[4 parts, 5 points each]** Compute the determinant of the following matrices.

(a) $\begin{bmatrix} 3 & 1 \\ -2 & -5 \end{bmatrix}$

(b) $\begin{bmatrix} 0 & 0 & 0 & a \\ b & 0 & c & d \\ e & f & g & h \\ i & 0 & j & k \end{bmatrix}$

(c) $\begin{bmatrix} 1 & 3 & -1 \\ 2 & 1 & -1 \\ 0 & 5 & -1 \end{bmatrix}$

(d) $\begin{bmatrix} 2 & 1 & 1 & 1 \\ 1 & 2 & 1 & 1 \\ 1 & 1 & 2 & 1 \\ 1 & 1 & 1 & 2 \end{bmatrix}$

4. **[5 points]** Let A be an $n \times n$ matrix such that $A^2 = I_n$. Prove that $\det(A)$ equals 1 or -1.

5. **[15 points]** Diagonalize the following matrix if possible by finding an invertible matrix P and a diagonal matrix D such that $A = PDP^{-1}$. There is no need to compute P^{-1}. If diagonalization is not possible, then explain why.

$$\begin{bmatrix} -5 & 8 & 0 \\ -4 & 7 & 0 \\ 8 & -16 & -1 \end{bmatrix}$$

6. **[15 points]** Give a formula for A^k, where $A = \begin{bmatrix} -4 & 3 \\ -6 & 5 \end{bmatrix}$.

7. [**5 parts, 3 points each**] True/False. Justify your answer.

   (a) If A and B are $n \times n$ matrices, then $\det(A+B) = \det(A) + \det(B)$.

   (b) Matrices A and B are similar if and only if A and B have the same eigenvalues with the same multiplicities.

   (c) Let A be a matrix with 7 rows and 10 columns. If the null space of A has dimension 4, then the column space of A has dimension 6.

   (d) If A is a square matrix and r is a scalar, then A is similar to rA.

   (e) If A and B are similar matrices, then A^{-1} and B^{-1} are also similar.

8. [**5 points**] Find a matrix B such that $B^2 = \begin{bmatrix} -1 & 5 \\ -10 & 14 \end{bmatrix}$.