Directions: You may work to solve these problems in groups, but all written work must be your own. Show all work; no credit for solutions without work.

1. [5.8.8] Let $A = \begin{bmatrix} 2 & 1 \\ 4 & 5 \end{bmatrix}$ and let $\mathbf{x}_0 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$. Execute the power method to generate $\mathbf{x}_k$ and μ_k for $k = 0, \ldots, 4$, keeping 3 decimal places. What is the estimated eigenvalue/eigenvector pair?

2. Let $\mathbf{u} = \begin{bmatrix} 1 & 2 & 3 & 4 \end{bmatrix}^T$ and $\mathbf{v} = \begin{bmatrix} 1 & -1 & 2 & -2 \end{bmatrix}^T$. Find a scalar α and a vector $\mathbf{w}$ such that $\mathbf{u} = \alpha\mathbf{v} + \mathbf{w}$ and $\mathbf{w}$ is orthogonal to $\mathbf{v}$.

3. [6.1.22] Let $\mathbf{u} = \begin{bmatrix} u_1 & u_2 & u_3 \end{bmatrix}^T$. Explain why $\mathbf{u} \cdot \mathbf{u} \geq 0$ directly from the definition of the dot product. When is $\mathbf{u} \cdot \mathbf{u} = 0$?

4. [6.1.24] Verify the *parallelogram law* for $\mathbf{u}$ and $\mathbf{v}$ in $\mathbb{R}^n$: $||\mathbf{u}+\mathbf{v}||^2 + ||\mathbf{u}-\mathbf{v}||^2 = 2||\mathbf{u}||^2 + 2||\mathbf{v}||^2$.

5. [6.1.19] True/False. Justify your answers.

   (a) $\mathbf{v} \cdot \mathbf{v} = ||\mathbf{v}||^2$

   (b) For any scalar c, $\mathbf{u} \cdot (c\mathbf{v}) = c(\mathbf{u} \cdot \mathbf{v})$.

   (c) If the distance from $\mathbf{u}$ to $\mathbf{v}$ equals the distance from $\mathbf{u}$ to $-\mathbf{v}$, then $\mathbf{u}$ and $\mathbf{v}$ are orthogonal.

   (d) For a square matrix A, vectors in Col$A$ are orthogonal to vectors in Nul$A$.

   (e) If vectors $\mathbf{v}_1, \ldots, \mathbf{v}_p$ span a subspace W and if $\mathbf{x}$ is orthogonal to each $\mathbf{v}_j$ for $j \in \{1, \ldots, p\}$, then $\mathbf{x}$ is in $W^\perp$.