Solutions

Directions:

1. Section: Math251 007
2. Write your name with one character in each box below.
3. Show all work. No credit for answers without work.

1. **[3 points]** Give parametric equations for the line through $(2, 1, -5)$ that is parallel to $\langle 1, 5, -1\rangle$.

$\vec{r} = \langle 2, 1, -5\rangle + t\langle 1, 5, -1\rangle$

$x(t) = 2 + t$
$y(t) = 1 + 5t$
$z(t) = -5 - t$

2. **[3 points]** Give a vector equation for the line at the intersection of planes $x+2y-5z = 5$ and $3x - 2y + z = -1$.

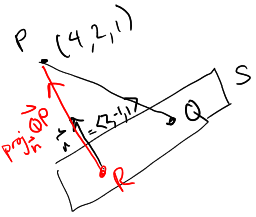
$$\begin{bmatrix} 1 & 2 & -5 & 5 \\ 3 & -2 & 1 & -1 \end{bmatrix} \xrightarrow{R2 \mathrel{+}= (-3)R1} \begin{bmatrix} 1 & 2 & -5 & 5 \\ 0 & -8 & 16 & -16 \end{bmatrix} \xrightarrow{R2 \div -\frac{1}{8}} \begin{bmatrix} 1 & 2 & -5 & 5 \\ 0 & 1 & -2 & 2 \end{bmatrix}$$

$$\xrightarrow{R1 \mathrel{+}= (-2)R2} \begin{bmatrix} x & y & z & \\ 1 & 0 & -1 & 1 \\ 0 & 1 & -2 & 2 \end{bmatrix}$$

$x = 1 + t$
$y = 2 + 2t$
$z = t$

So $\vec{r} = \langle 1, 2, 0\rangle + t\langle 1, 2, 1\rangle$

3. Let S be the plane $2x - y + z = 1$ and let P be the point $(4, 2, 1)$.



(a) **[3 points]** Find the distance between S and P.

Find base point Q on S: take $Q = (0, 0, 1)$. Now $\vec{QP} = \langle 4, 2, 0\rangle$

$$\text{Distance} = \left|\text{proj}_{\vec{n}} \vec{QP}\right| = \left|\frac{\vec{QP}\cdot\vec{n}}{|\vec{n}|^2}\vec{n}\right| = \frac{|\vec{QP}\cdot\vec{n}|}{|\vec{n}|} = \frac{|\langle 4,2,0\rangle\cdot\langle 2,-1,1\rangle|}{|\langle 2,-1,1\rangle|}$$

$$= \frac{|(4)(2) + (2)(-1) + (0)(1)|}{\sqrt{2^2 + (-1)^2 + 1^2}} = \frac{|6|}{\sqrt{4+2}} = \frac{6}{\sqrt{6}} = \boxed{\sqrt{6}}$$

(b) **[1 point]** Find the point on S that is closest to P.

$$\text{Closest Point} = P - \text{proj}_{\vec{n}} \vec{QP} = (4,2,1) - \frac{\vec{QP}\cdot\vec{n}}{\vec{n}\cdot\vec{n}}\vec{n} = (4,2,1) - \frac{6}{6}\vec{n} = (4,2,1) - (2,-1,1)$$

$$= \boxed{(2, 3, 0)}.$$