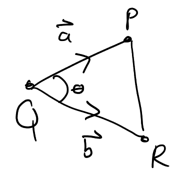
Directions:

1. Section: Math251-007
2. Write your name with one character in each box below.
3. Show all work. No credit for answers without work.

1. **[3 points]** Let $P(2,-1,-2)$, $Q(3,0,3)$ and $R(1,2,-1)$ be points in $\mathbb{R}^3$. Find the angle formed by the triangle PQR at the vertex Q. Leave your answer in terms of inverse trig functions.

$$\begin{array}{r} 27 \\ \times\, 24 \\ \hline 108 \\ 540 \\ \hline 648 \end{array}$$



$\vec{a} = \vec{QP} = \langle 2-3, -1-0, -2-3\rangle = \langle -1,-1,-5\rangle$

$\vec{b} = \vec{QR} = \langle 1-3, 2-0, -1-3\rangle = \langle -2, 2, -4\rangle$

$\vec{a}\cdot\vec{b} = |\vec{a}||\vec{b}|\cos\theta,$

$$\cos\theta = \frac{\vec{a}\cdot\vec{b}}{|\vec{a}||\vec{b}|} = \frac{(-1)(-2)+(-1)(2)+(-5)(-4)}{\sqrt{1+1+25}\cdot\sqrt{4+4+16}} = \frac{20}{\sqrt{27}\cdot\sqrt{24}} = \frac{20}{3\sqrt{3}\cdot 2\sqrt{6}} = \frac{10}{3\sqrt{9\cdot 2}} = \frac{10}{9\sqrt{2}}$$

So $\theta = \boxed{\cos^{-1}\frac{10}{9\sqrt{2}}}$

2. **[2 parts, 3 points each]** Let $\mathbf{a} = \langle 1,-1,3\rangle$ and $\mathbf{b} = \langle 5,1,-2\rangle$.

   (a) Find $\mathbf{a}\times\mathbf{b}$.

$$\vec{a}\times\vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -1 & 3 \\ 5 & 1 & -2 \end{vmatrix} = \hat{i}\begin{vmatrix} -1 & 3 \\ 1 & -2 \end{vmatrix} - \hat{j}\begin{vmatrix} 1 & 3 \\ 5 & -2 \end{vmatrix} + \hat{k}\begin{vmatrix} 1 & -1 \\ 5 & 1 \end{vmatrix} = -\hat{i} + 17\hat{j} + 6\hat{k}$$

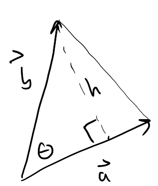
$$= \boxed{\langle -1, 17, 6\rangle.}$$

$(17)^2 = (10+7)^2 = 100 + 2\cdot 70 + 49 = 289$

   (b) Find the area of the triangle formed by placing the tails of the vectors $\mathbf{a}$ and $\mathbf{b}$ at the origin.

$$\text{area}(T) = \frac{1}{2}|\vec{a}\times\vec{b}| = \frac{1}{2}\sqrt{(-1)^2+(17)^2+(6)^2} = \frac{1}{2}\sqrt{1+289+36} = \boxed{\frac{1}{2}\sqrt{326}}$$

3. **[1 point]** Let $\mathbf{a},\mathbf{b}\in\mathbb{R}^n$ and let T be the triangle with vertices $\mathbf{0},\mathbf{a},\mathbf{b}$. The cross product is defined only in 3-dimensions, so it cannot be used to compute the area of T. Use the dot product theorem and the formula $\sin\theta = \sqrt{1-\cos^2\theta}$ for $0\le\theta\le\pi$ to find a formula for the area of T in terms of the magnitudes of $\mathbf{a}$ and $\mathbf{b}$ and the dot product $\mathbf{a}\cdot\mathbf{b}$.



$$\text{Area}(T) = \frac{1}{2}|\vec{a}|\cdot h = \frac{1}{2}|\vec{a}||\vec{b}|\sin\theta = \frac{1}{2}|\vec{a}||\vec{b}|\sqrt{1-\cos^2\theta}$$

$$= \frac{1}{2}|\vec{a}||\vec{b}|\sqrt{1 - \frac{(\vec{a}\cdot\vec{b})^2}{|\vec{a}|^2|\vec{b}|^2}}$$

$$= \boxed{\frac{1}{2}\sqrt{|\vec{a}|^2|\vec{b}|^2 - (\vec{a}\cdot\vec{b})^2}}$$

$\sin\theta = \frac{h}{|\vec{b}|}$

$\vec{a}\cdot\vec{b} = |\vec{a}||\vec{b}|\cos\theta$