Directions:

1. Section: Math251 007
2. Write your name with one character in each box below.
3. Show all work. No credit for answers without work.
4. This assessment is closed book and closed notes. You may not use electronic devices, including calculators, laptops, and cell phones.

Academic Integrity Statement: I will complete this work on my own without assistance, knowing or otherwise, from anyone or anything other than the instructor. I will not use any electronic equipment or notes (except as permitted by an existing official, WVU-authorized accommodation).

Signature: Solutions

1. Let $\mathbf{F} = \langle -4y, x\rangle$ and let $g(x, y) = \frac{x^2}{4} + y^2$.

   (a) **[2 points]** Show that the outputs of $\mathbf{F}$ are tangent to the level curves of g.

   We have $\nabla g = \langle g_x, g_y \rangle = \langle \frac{x}{2}, 2y \rangle$.

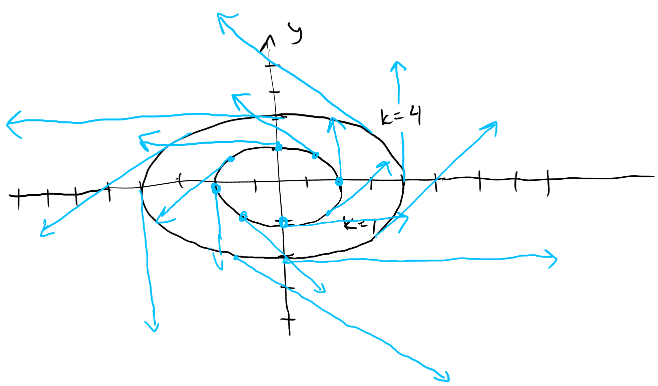
   We compute $F \cdot \nabla g = \langle -4y, x \rangle \cdot \langle \frac{x}{2}, 2y \rangle = -2xy + 2xy = 0$.

   Since F is orthogonal to ∇g, the output vectors of F are tangent to the level curves of g.

   (b) **[3 points]** Sketch sample level curves of g and the vector field $\mathbf{F}$.

   $\frac{x^2}{4} + y^2 = k$

   $\frac{x^2}{(2\sqrt{k})^2} + \frac{y^2}{(\sqrt{k})^2}$



   (c) **[1 point]** Is g a potential function for $\mathbf{F}$? Explain why or why not.

   No, since $\nabla g = \langle \frac{x}{2}, 2y \rangle \neq \langle -4y, x \rangle = F$, we see that g is <u>not</u> a potential function for $\vec{F}$.

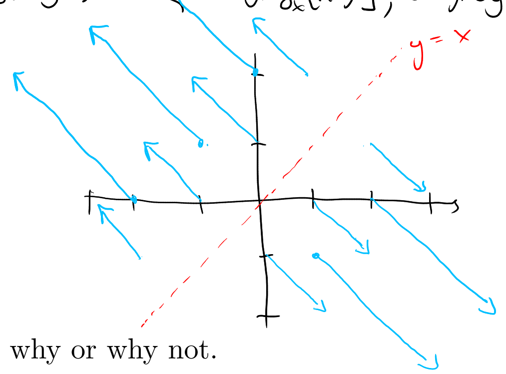
2. Let $f(x, y) = \frac{1}{2}(x - y)^2$.

   (a) **[3 points]** Find the gradient vector field $\mathbf{F}$ of f and sketch it.

   $\nabla f = \langle f_x, f_y \rangle = \langle \frac{\partial}{\partial x}\left[\frac{1}{2}(x-y)^2\right], \frac{\partial}{\partial y}\left[\frac{1}{2}(x-y)^2\right] \rangle = \langle (x-y)\frac{\partial}{\partial x}[x-y], (x-y)\frac{\partial}{\partial y}[x-y] \rangle$

   $= \langle (x-y)\cdot 1, (x-y)\cdot(-1) \rangle = \langle x-y, y-x \rangle$.

   $\cdot \langle x, y \rangle + \langle x-y, y-x \rangle = \langle 2x-y, 2y-x \rangle$



   (b) **[1 point]** Is $\mathbf{F}$ a conservative vector field? Explain why or why not.

   Yes, since there exists a potential function f such that $\nabla f = F$, we have that $\boxed{F \text{ is conservative}}$.