Directions:

1. Write your name with one character in each box below.
2. Show all work. No credit for answers without work.

1. **[4 parts, 1 point each]** Compute the following matrices if defined.

$$A = \begin{bmatrix} 1 & -1 \\ 2 & 4 \end{bmatrix} \quad B = \begin{bmatrix} 2 & 0 \\ 0 & 3 \\ 1 & -3 \end{bmatrix} \quad C = \begin{bmatrix} 3 & -2 \\ 1 & 5 \end{bmatrix} \quad D = \begin{bmatrix} -1 & 2 & 5 \end{bmatrix}$$

(a) $A + 2C$

$$\begin{bmatrix} 1 & -1 \\ 2 & 4 \end{bmatrix} + 2\begin{bmatrix} 3 & -2 \\ 1 & 5 \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ 2 & 4 \end{bmatrix} + \begin{bmatrix} 6 & -4 \\ 2 & 10 \end{bmatrix} = \boxed{\begin{bmatrix} 7 & -5 \\ 4 & 14 \end{bmatrix}}$$

(b) BC

$$\begin{bmatrix} 2 & 0 \\ 0 & 3 \\ 1 & -3 \end{bmatrix}\begin{bmatrix} 3 & -2 \\ 1 & 5 \end{bmatrix} = \boxed{\begin{bmatrix} 6 & -4 \\ 3 & 15 \\ 0 & -17 \end{bmatrix}}$$

(c) DD^T

$$\begin{bmatrix} -1 & 2 & 5 \end{bmatrix}\begin{bmatrix} -1 \\ 2 \\ 5 \end{bmatrix} = \begin{bmatrix} (-1)^2 + 2^2 + 5^2 \end{bmatrix} = \boxed{\begin{bmatrix} 30 \end{bmatrix}}$$

(d) $D^T D$

$$\begin{bmatrix} -1 \\ 2 \\ 5 \end{bmatrix}\begin{bmatrix} -1 & 2 & 5 \end{bmatrix} = \boxed{\begin{bmatrix} 1 & -2 & -5 \\ -2 & 4 & 10 \\ -5 & 10 & 25 \end{bmatrix}}$$

2. **[6 points]** Solve the following system.

$$\begin{aligned} -3x_1 + 4x_2 - 4x_3 &= 1 \\ -11x_1 + 16x_2 - 15x_3 &= 2 \\ -x_1 + x_2 - x_3 &= 3 \end{aligned}$$

$$\begin{bmatrix} -3 & 4 & -4 & 1 \\ -11 & 16 & -15 & 2 \\ -1 & 1 & -1 & 3 \end{bmatrix} \xrightarrow[R1 \leftrightarrow R3]{R3 \cdot (-1)} \begin{bmatrix} 1 & -1 & 1 & -3 \\ -11 & 16 & -15 & 2 \\ -3 & 4 & -4 & 1 \end{bmatrix} \xrightarrow[R3 \leftarrow R3 + (3)R1]{R2 \leftarrow R2 + (11)R1} \begin{bmatrix} 1 & -1 & 1 & -3 \\ 0 & 5 & -4 & -31 \\ 0 & 1 & -1 & -8 \end{bmatrix}$$

$$\xrightarrow{R2 \leftrightarrow R3} \begin{bmatrix} 1 & -1 & 1 & -3 \\ 0 & 1 & -1 & -8 \\ 0 & 5 & -4 & -31 \end{bmatrix} \xrightarrow{R3 \leftarrow R3 + (-5)R2} \begin{bmatrix} 1 & -1 & 1 & -3 \\ 0 & 1 & -1 & -8 \\ 0 & 0 & 1 & 9 \end{bmatrix} \xrightarrow[R2 \leftarrow R2 + R3]{R1 \leftarrow R1 + (-1)R3} \begin{bmatrix} 1 & -1 & 0 & -12 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 9 \end{bmatrix}$$

$$\xrightarrow{R1 \leftarrow R1 + R2} \begin{bmatrix} 1 & 0 & 0 & -11 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 9 \end{bmatrix}$$

So $\boxed{x_1 = -11, \ x_2 = 1, \ x_3 = 9.}$ is the unique solution.